General Relativistic Mean Field Theory for Rotating Nuclei

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(February 9, 2008)

Abstract

We formulate a general relativistic mean field theory for rotating nuclei starting from the special relativistic σ-ω model Lagrangian. The tetrad formalism is adopted to generalize the model to the accelerated frame.

Quantum hadrodynamics(QHD) is a quantum field theory which treats nucleons and mesons as elementary degrees of freedom. The origin of relativistic nuclear models can be traced back to the work of Duerr [1] who reformulated a non-relativistic field theoretical nuclear model of Johnson and Teller [2]. The present form of QHD was established by Chin and Walecka who reproduced the saturation property of the nuclear matter within the mean field approximation in the 70's [3]. Since then it has been enjoying its success in accounting for various nuclear phenomena [4]. Presently it is appreciated as a reliable way, alternative to traditional non-relativistic nuclear theories such as the Skyrme-Hartree-Fock calculation, of describing not only the ground state properties of finite(spherical [5], deformed [6] and superdeformed [7]) nuclei but also the scattering observables [8]. Recently it has also been applied extensively to exotic nuclei [9]. Incorporating the polarization of the Fermi and Dirac sea, which is neglected in the mean field approximation, on the other hand, QHD can be an effective theory for the hadron properties such as masses of vector mesons in finite density nuclear medium [10] to which lattice QCD calculations have not been available.

Two directions of extensions of the relativistic mean field theory to the description of the excited states have been done so far. One is to the giant resonances [11] and the other is to the yrast states of rotating nuclei [12,13]. In [13], Münich group first applied this model to the yrast states of ^{20}Ne and got similar results to the Skyrme-Hartree-Fock calculation. In some following papers [14], they showed that this model could also reproduce the moments of inertia of medium heavy and heavy superdeformed nuclei in which effects of the pairing correlation was assumed to be not important. This model was extended to the non-uniform three-dimensional rotation and a method of its quantization was also discussed [15]. From the theoretical point of view, however, Koepf and Ring's formulation based on the Lorentz transformation is not adequate because the rotating frame is an accelerated one, and the coordinate transformation from the laboratory frame to the rotating one is not a Lorentz but a general coordinate transformation. The main reason why we adopt QHD, a special relativistic model, is to respect the Lorentz covariance even if velocities involved are not so large. Parallel to this, we should adopt general relativistic models for the phenomena for

which general coordinate transformations are necessary even if the curvature of the space-time is zero. Therefore, in this Letter, we develop a general relativistic mean field theory for rotating nuclei by adopting the tetrad formalism [16].

The crucial point is the transformation property of the nucleon field under the coordinate transformation

$$x^\alpha = \begin{pmatrix} t \\ \boldsymbol{x} \end{pmatrix} \rightarrow \tilde{x}^\mu = \begin{pmatrix} \tilde{t} \\ \tilde{\boldsymbol{x}} \end{pmatrix} = \begin{pmatrix} 1 & \mathbf{0}^T \\ \mathbf{0} & R_x(t) \end{pmatrix} \begin{pmatrix} t \\ \boldsymbol{x} \end{pmatrix}, \tag{1}$$

$$R_x(t) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\Omega t & \sin\Omega t \\ 0 & -\sin\Omega t & \cos\Omega t \end{pmatrix}, \tag{2}$$

where x^α stands for the laboratory frame while $\tilde{x}^\mu$ a frame uniformly rotating around the $\tilde{x} = x$ axis with an angular velocity Ω. Koepf and Ring adopted

$$\psi(x) \rightarrow \tilde{\psi}(\tilde{x}) = e^{i\Omega t \Sigma_x}\psi(x),\ \Sigma_x = \frac{1}{4}[\gamma^2, \gamma^3], \tag{3}$$

which is applicable only to the constant angle shift. But obviously this is not for the present case. Therefore we have to adopt

$$\psi(x) \rightarrow \tilde{\psi}(\tilde{x}) = \psi(x), \tag{3'}$$

as known in the quantum theory of gravity [16]; the fermion field transforms as a scalar under the general coordinate transformation. The tetrad formalism gives us how to treat the spinor field in general relativity. In the following, we develop our formalism.

First we consider a non-inertial frame (either curved or flat) represented by a metric tensor $g_{\mu\nu}(\tilde{x})$. The principle of equivalence allows us to construct a set of coordinates $\xi^\alpha_X(\tilde{x})$ that are locally inertial at $\tilde{x}^\mu = X^\mu$. Then the metric tensors of the non-inertial and the inertial frames are related as

$$g_{\mu\nu}(\tilde{x}) = V^\alpha_{\ \mu}(\tilde{x})V^\beta_{\ \nu}(\tilde{x})\eta_{\alpha\beta}, \tag{4}$$

here a tetrad is defined by

$$V^{\alpha}_{\ \mu}(X) = \left(\frac{\partial \xi^{\alpha}_{X}(\tilde{x})}{\partial \tilde{x}^{\mu}}\right)_{\tilde{x}=X}. \tag{5}$$

Labels $\alpha, \beta, \cdots$ refer to the inertial frames while $\mu, \nu, \cdots$ to the non-inertial ones. This quantity, the tetrad, transforms as a vector not only under the general coordinate transformation,

$$V^{\alpha}_{\ \mu}(\tilde{x}) \to V'^{\alpha}_{\ \mu}(\tilde{x}') = \frac{\partial \tilde{x}^{\nu}}{\partial \tilde{x}'^{\mu}} V^{\alpha}_{\ \nu}(\tilde{x}), \tag{6}$$

but also under the local Lorentz transformation,

$$V^{\alpha}_{\ \mu}(X) \to V'^{\alpha}_{\ \mu}(X) = \Lambda^{\alpha}_{\ \beta}(X) V^{\beta}_{\ \mu}(X). \tag{7}$$

The latter which leaves eq.(4) invariant allows us to choose various forms for $V^{\alpha}_{\ \mu}$. We will make use of this property later. The main advantage of introducing the tetrad is that any tensors $B^{\mu\nu\cdots}(\tilde{x})$ with respect to the general coordinate transformation can be converted to scalars with respect to it, which is at the same time tensors with respect to the local Lorentz transformation,

$$B^{\mu\nu\cdots}(\tilde{x}) \Longrightarrow {}^{*}B^{\alpha\beta\cdots}(\tilde{x}) = V^{\alpha}_{\ \mu}(\tilde{x}) V^{\beta}_{\ \nu}(\tilde{x}) \cdots B^{\mu\nu\cdots}(\tilde{x}) \tag{8}$$

: general coordinate scalar and Lorentz tensor,

by contracting with the tetrad. This implies that this contraction is not necessary for the general coordinate scalar quantities,

$$\phi(\tilde{x}) \Longrightarrow {}^{*}\phi(\tilde{x}) = \phi(\tilde{x}) \tag{9}$$

: general coordinate scalar and Lorentz scalar,

$$\psi(\tilde{x}) \Longrightarrow {}^{*}\psi(\tilde{x}) = \psi(\tilde{x}) \tag{10}$$

: general coordinate scalar and Lorentz spinor.

The covariant derivative with respect to the local Lorentz transformation of the general coordinate scalar ${}^{*}\varphi(={}^{*}\psi, {}^{*}\phi, {}^{*}A^{\alpha}, \cdots)$ is given by [16]

$$\tilde{\nabla}_{\alpha} {}^{*}\varphi = V_{\alpha}^{\ \mu} \tilde{\nabla}_{\mu} {}^{*}\varphi \equiv V_{\alpha}^{\ \mu}(\tilde{\partial}_{\mu} + \Gamma_{\mu}){}^{*}\varphi, \tag{11}$$

with the connection

$$\Gamma_\mu(\tilde{x}) = \frac{1}{2}\sigma^{\alpha\beta}V_\alpha^{\ \nu}(\tilde{x})V_{\beta\nu;\mu}(\tilde{x}), \tag{12}$$

where $\sigma^{\alpha\beta}$ is the generator of the Lorentz group, the symbol $;\mu$ denotes the well-known covariant derivative with respect to the general coordinate transformation.

Collecting all the ingredients given above, we can generalize the Lagrangian to the non-inertial frame with the following prescriptions:

1) Write the Lagrangian in the Minkowski space-time.

2) Contract all the tensors with the tetrad.

3) Replace all the derivatives with the covariant derivatives.

4) Multiply $\sqrt{-g}$ to cast the resulting quantity into a scalar density with respect to the general coordinate transformation. Here

$$g = \det(g_{\mu\nu}),\ \sqrt{-g} = \det(V^{\alpha}_{\ \mu}). \tag{13}$$

The results for the σ-ω model,

$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_\sigma + \mathcal{L}_\omega + \mathcal{L}_{\text{int}}, \tag{14}$$

$$\mathcal{L}_N = \overline{\psi}(i\gamma^\alpha\partial_\alpha - M)\psi, \tag{15}$$

$$\mathcal{L}_\sigma = \frac{1}{2}(\partial_\alpha\sigma)(\partial^\alpha\sigma) - \frac{1}{2}m_\sigma^2\sigma^2, \tag{16}$$

$$\mathcal{L}_\omega = -\frac{1}{4}F_{\alpha\beta}F^{\alpha\beta} + \frac{1}{2}m_\omega^2\omega_\alpha\omega^\alpha, \tag{17}$$

$$\mathcal{L}_{\text{int}} = g_\sigma\overline{\psi}\psi\sigma - g_\omega\overline{\psi}\gamma^\alpha\psi\omega_\alpha, \tag{18}$$

are

$$\mathcal{L}_N \rightarrow$$

$$\sqrt{-g}\left[\overline{\psi}(\tilde{x})(i\tilde{\gamma}^\mu(\tilde{x})(\tilde{\partial}_\mu + \Gamma_\mu(\tilde{x})) - M)\psi(\tilde{x})\right], \tag{19}$$

with eq.(12),

$$
\begin{aligned}
\mathcal{L}_\sigma \;\to\;& \\
&\sqrt{-g}\Big[\frac{1}{2}V_\alpha^{\ \mu}(\tilde{x})(\tilde{\nabla}_\mu\sigma(\tilde{x}))V^\alpha_{\ \nu}(\tilde{x})(\tilde{\nabla}^\nu\sigma(\tilde{x})) - \frac{1}{2}m_\sigma^2\sigma^2(\tilde{x})\Big] \\
=& \sqrt{-g}\Big[\frac{1}{2}(\tilde{\partial}_\mu\sigma(\tilde{x}))(\tilde{\partial}^\mu\sigma(\tilde{x})) - \frac{1}{2}m_\sigma^2\sigma^2(\tilde{x})\Big],
\end{aligned}
\tag{20}
$$

owing to the fact that the covariant derivative coincides with the ordinary one for the scalar field,

$$
\begin{aligned}
\mathcal{L}_\omega \;\to\;& \\
&\sqrt{-g}\Big[-\frac{1}{4}\left(V_\alpha^{\ \mu}(\tilde{x})\tilde{\nabla}_\mu(V_\beta^{\ \nu}(\tilde{x})\omega_\nu(\tilde{x})) - V_\beta^{\ \mu}(\tilde{x})\tilde{\nabla}_\mu(V_\alpha^{\ \nu}(\tilde{x})\omega_\nu(\tilde{x}))\right) \\
&\quad \times \left(V^\alpha_{\ \mu}(\tilde{x})\tilde{\nabla}^\mu(V^\beta_{\ \nu}(\tilde{x})\omega^\nu(\tilde{x})) - V^\beta_{\ \mu}(\tilde{x})\tilde{\nabla}^\mu(V^\alpha_{\ \nu}(\tilde{x})\omega^\nu(\tilde{x}))\right) \\
&\quad + \frac{1}{2}m_\omega^2 V_\alpha^{\ \mu}(\tilde{x})\omega_\mu(\tilde{x})V^\alpha_{\ \nu}(\tilde{x})\omega^\nu(\tilde{x})\Big] \\
=& \sqrt{-g}\Big[-\frac{1}{4}F_{\mu\nu}(\tilde{x})F^{\mu\nu}(\tilde{x}) + \frac{1}{2}m_\omega^2\omega_\mu(\tilde{x})\omega^\mu(\tilde{x})\Big],
\end{aligned}
\tag{21}
$$

here

$$
F_{\mu\nu}(\tilde{x}) = \omega_{\nu;\mu}(\tilde{x}) - \omega_{\mu;\nu}(\tilde{x}) = \tilde{\partial}_\mu\omega_\nu(\tilde{x}) - \tilde{\partial}_\nu\omega_\mu(\tilde{x}),
\tag{22}
$$

and

$$
\begin{aligned}
&\mathcal{L}_{\text{int}} \to \\
&\sqrt{-g}\left[g_\sigma\overline{\psi}(\tilde{x})\psi(\tilde{x})\sigma(\tilde{x}) - g_\omega\overline{\psi}(\tilde{x})\tilde{\gamma}^\mu(\tilde{x})\psi(\tilde{x})\omega_\mu(\tilde{x})\right].
\end{aligned}
\tag{23}
$$

In (19) and (23), the generalized γ matrices are defined as

$$
\tilde{\gamma}^\mu(\tilde{x}) = \gamma^\alpha V_\alpha^{\ \mu}(\tilde{x}),
\tag{24}
$$

and they satisfy

$$
\{\tilde{\gamma}^\mu(\tilde{x}), \tilde{\gamma}^\nu(\tilde{x})\} = 2g^{\mu\nu}(\tilde{x}).
\tag{25}
$$

The variational principle applied to the above generalized Lagrangian for the non-inertial frame gives the equations of motion:

$$\Big[i\tilde{\gamma}^{\mu}(\tilde{x})(\tilde{\partial}_{\mu}+\Gamma_{\mu}(\tilde{x}))-M+g_{\sigma}\sigma(\tilde{x})$$
$$-g_{\omega}\tilde{\gamma}^{\mu}(\tilde{x})\omega_{\mu}(\tilde{x})\big]\,\psi(\tilde{x})=0, \tag{26}$$

$$\tilde{\partial}_{\mu}\left[\tilde{\partial}^{\mu}\sigma(\tilde{x})\right]+m_{\sigma}^{2}\sigma(\tilde{x})=g_{\sigma}\overline{\psi}(\tilde{x})\psi(\tilde{x}), \tag{27}$$

and

$$F^{\mu\nu}{}_{;\mu}(\tilde{x})+m_{\omega}\omega^{\nu}(\tilde{x})=g_{\omega}\overline{\psi}(\tilde{x})\tilde{\gamma}^{\nu}(\tilde{x})\psi(\tilde{x}), \tag{28}$$

for the nucleon, σ meson, and ω meson, respectively.

Now we choose a specific flat but non-inertial frame, that is a uniformly rotating frame given by the coordinate transformation (1). The metric tensor in this case is

$$\begin{aligned} g_{\mu\nu}(\tilde{x}) &= \frac{\partial x^{\alpha}}{\partial\tilde{x}^{\mu}}\frac{\partial x^{\beta}}{\partial\tilde{x}^{\nu}}\eta_{\alpha\beta} \\ &= \begin{pmatrix} 1-\Omega^{2}(\tilde{y}^{2}+\tilde{z}^{2}) & 0 & \Omega\tilde{z} & -\Omega\tilde{y} \\ 0 & -1 & 0 & 0 \\ \Omega\tilde{z} & 0 & -1 & 0 \\ -\Omega\tilde{y} & 0 & 0 & -1 \end{pmatrix}. \end{aligned} \tag{29}$$

Looking at eqs.(4),(5), and (29), the simplest choice of the tetrad is

$$\begin{aligned} V^{\alpha}_{\ \mu}(\tilde{x}) &= \frac{\partial x^{\alpha}}{\partial\tilde{x}^{\mu}} \\ &= \begin{pmatrix} 1 & \mathbf{0}^{T} \\ 0 & \\ -\Omega(\tilde{y}\sin\Omega\tilde{t}+\tilde{z}\cos\Omega\tilde{t}) & R_{x}^{T}(\tilde{t}) \\ \Omega(\tilde{y}\cos\Omega\tilde{t}-\tilde{z}\sin\Omega\tilde{t}) & \end{pmatrix}, \end{aligned} \tag{30}$$

i.e., the choice of the inertial coordinate

$$\xi^{\alpha}_{X}(\tilde{x})=x^{\alpha}. \tag{31}$$

We call this the *fundamental choice*. This choice results in

$$\Gamma_\mu(\tilde{x}) = 0, \tag{32}$$

for the spinor field, and accordingly

$$\Big[(R_x(\tilde{t})\boldsymbol{\alpha})\cdot(\frac{1}{i}\tilde{\boldsymbol{\nabla}} - g_\omega\tilde{\boldsymbol{\omega}}(\tilde{x})) + \beta(M - g_\sigma\sigma(\tilde{x}))$$

$$+ g_\omega\tilde{\omega}^0(\tilde{x}) - \Omega\tilde{L}_x\Big]\,\psi_i(\tilde{x}) = i\tilde{\partial}_0\psi_i(\tilde{x}), \tag{33}$$

$$\Big[\tilde{\partial}_0^2 - \tilde{\boldsymbol{\nabla}}^2 + m_\sigma^2 - \Omega^2\tilde{L}_x^2\Big]\,\sigma(\tilde{x}) = g_\sigma\rho_s(\tilde{x}), \tag{34}$$

$$\Big[\tilde{\partial}_0^2 - \tilde{\boldsymbol{\nabla}}^2 + m_\omega^2 - \Omega^2\tilde{L}_x^2\Big]\,\tilde{\omega}^0(\tilde{x}) = g_\omega\tilde{\rho}_v(\tilde{x}), \tag{35}$$

$$\Big[\tilde{\partial}_0^2 - \tilde{\boldsymbol{\nabla}}^2 + m_\omega^2 - \Omega^2(\tilde{L}_x + S_x)^2\Big]\,\tilde{\boldsymbol{\omega}}(\tilde{x}) = g_\omega\tilde{\boldsymbol{j}}_v(\tilde{x}), \tag{36}$$

for the equations of motion within the mean field approximation where the nucleon field ψ is expanded in terms of single particle states ψ_is. Here

$$\rho_s(\tilde{x}) = \sum_i^{\rm occ} \overline{\psi}_i(\tilde{x})\psi_i(\tilde{x}), \tag{37}$$

$$\rho_v(\tilde{x}) = \sum_i^{\rm occ} \psi_i^\dagger(\tilde{x})\psi_i(\tilde{x}), \tag{38}$$

$$\boldsymbol{j}_v(\tilde{x}) = \sum_i^{\rm occ} \psi_i^\dagger(\tilde{x})\boldsymbol{\alpha}\psi_i(\tilde{x}), \tag{39}$$

and a similar redefinition of the vector quantities according to Koepf and Ring,

$$\tilde{\omega}^0 = \omega^0,\ \tilde{\boldsymbol{\omega}} = \boldsymbol{\omega} - (\boldsymbol{\Omega}\times\tilde{\boldsymbol{x}})\omega^0, \tag{40a}$$

$$\tilde{\rho}_v = \rho_v,\ \tilde{\boldsymbol{j}}_v = R_x(\tilde{t})\boldsymbol{j}_v - (\boldsymbol{\Omega}\times\tilde{\boldsymbol{x}})\rho_v, \tag{40b}$$

was done. Note that S_x is the ordinary spin operator for the spin=1 field,

$$S_x = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}. \tag{41}$$

Obviously eq.(33) is not stationary in the sense of the ordinary cranking model because $R_x(\tilde{t})$ in the first term is time dependent.

Therefore another choice of the tetrad is desirable to formulate a stationary mean field theory parallel to the traditional non-relativistic cranking model. This is possible by making

use of the degrees of freedom of the local Lorentz transformation (7). This is due to the fact that $V^{\alpha}_{\ \mu}$ has 16 components whereas only 10 components are independent. Utilizing this freedom of choosing 6 components, we adopt another form for the tetrad,

$$V^{\alpha}_{\ \mu}(\tilde{x}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\Omega\tilde{z} & 0 & 1 & 0 \\ \Omega\tilde{y} & 0 & 0 & 1 \end{pmatrix}, \tag{30'}$$

which is called the *canonical choice* [17]. In the present case, this corresponds to choosing the 'instantaneously rest frame' with respect to the rotating one as the locally inertial frame. This choice results in

$$\Gamma_{\mu}(\tilde{x}) = \begin{pmatrix} -i\Omega\Sigma_x \\ \mathbf{0} \end{pmatrix}, \tag{32'}$$

for the spinor field, and accordingly

$$\Big[\boldsymbol{\alpha}\cdot(\frac{1}{i}\tilde{\boldsymbol{\nabla}} - g_\omega\tilde{\boldsymbol{\omega}}(\tilde{x})) + \beta(M - g_\sigma\sigma(\tilde{x})) + g_\omega\tilde{\omega}^0(\tilde{x}) - \Omega(\tilde{L}_x + \Sigma_x)\Big]\psi_i(\tilde{x}) = i\tilde{\partial}_0\psi_i(\tilde{x}), \tag{33'}$$

$$\Big[\tilde{\partial}_0^2 - \tilde{\boldsymbol{\nabla}}^2 + m_\sigma^2 - \Omega^2\tilde{L}_x^2\Big]\sigma(\tilde{x}) = g_\sigma\rho_s(\tilde{x}), \tag{34'}$$

$$\Big[\tilde{\partial}_0^2 - \tilde{\boldsymbol{\nabla}}^2 + m_\omega^2 - \Omega^2\tilde{L}_x^2\Big]\tilde{\omega}^0(\tilde{x}) = g_\omega\tilde{\rho}_v(\tilde{x}), \tag{35'}$$

$$\Big[\tilde{\partial}_0^2 - \tilde{\boldsymbol{\nabla}}^2 + m_\omega^2 - \Omega^2(\tilde{L}_x + S_x)^2\Big]\tilde{\boldsymbol{\omega}}(\tilde{x}) = g_\omega\tilde{\boldsymbol{j}}_v(\tilde{x}), \tag{36'}$$

for the equations of motion. Eq.(40b) is replaced by

$$\tilde{\rho}_v = \rho_v,\ \tilde{\boldsymbol{j}}_v = \boldsymbol{j}_v - (\boldsymbol{\Omega}\times\tilde{\boldsymbol{x}})\rho_v, \tag{40b'}$$

while (40a) is independent of the choice of the tetrad.

Assuming the time dependence

$$\psi_i(\tilde{t},\tilde{\boldsymbol{x}}) = \psi_i(\tilde{\boldsymbol{x}})e^{-i\tilde{e}_i\tilde{t}}, \tag{42}$$

where $\tilde{e}_i$ is the single-particle routhian, and the usual $\tilde{t}$-independence of the meson mean fields, we come to the desired stationary theory. This coincides with Koepf and Ring's. The total energy in the laboratory frame, $\int d^3xT^{00}$, can be calculated from the energy-momentum tensor in the rotating frame, $\tilde{T}^{\mu\nu}(\tilde{x})$, given by the tetrad formalism [16]. Again the result coincides with theirs.

The reason why they obtained the correct expressions starting from eq.(3) is clear. Since they defined the transformation property of the γ matrices such that $\overline{\psi}\gamma^\alpha\psi$ transforms as a (Lorentz) vector, their transformed γ matrices absorbed the inadequateness of the transformation property of the fermion field. In addition, their transformation (3) for the spinor and that for the γ matrices can be regarded as simulating the local Lorentz transformation from the fundamental to the canonical tetrad. These implications are clarified by constructing a correct general relativistic formulation.

To summarize, we have formulated a general relativistic mean field theory for rotating nuclei adopting the tetrad formalism. We applied this formulation to the σ-ω model which has been known to give good descriptions of various nuclear phenomena. We needed to adopt the so-called canonical choice of the tetrad to obtain a stationary equation of motion in the sense of the ordinary non-relativistic cranking model. The results are the same as those of Koepf and Ring who started from a special relativistic transformation property; their inadequateness was absorbed by their transformed γ matrices [13].

A possible way to go beyond the mean field approximation is the method of Kaneko, Nakano and one of the present authors [15] but any numerical application along this way has not been done. A systematic numerical calculation of the yrast states of not only stable but also unstable nuclei based on the present mean field theory is under progress and will be published separately.

We acknowledge the comments of K.Harada which inspired the present study. Discussions with R.R.Hilton and J.König are also acknowledged. This work was supported in part by the Grant-in-Aid for Scientific Research from the Ministry of Education, Science and Culture

(No.08740209).

REFERENCES

[1] H.-P.Duerr, Phys. Rev. **103**, 469 (1956).

[2] M.H.Johnson and E.Teller, Phys. Rev. **98**, 783 (1955).

[3] S.A.Chin and J.D.Walecka, Phys. Lett. B **52**, 24 (1974); J.D.Walecka, Ann. Phys. **83**, 491 (1974); S.A.Chin, ibid. **108**, 301 (1977).

[4] As recent reviews, P.Ring, Prog. Part. Nucl. Phys. **37**, 193 (1996); B.D.Serot and J.D.Walecka, preprint(nucl-th/9701058).

[5] Y.K.Gambhir, P.Ring and A.Thimet, Ann. Phys. **198**, 132 (1990).

[6] C.E.Price and G.E.Walker, Phys. Rev. C **36**, 354 (1987); R.J.Fuhnstahl, C.E.Price and G.E.Walker, ibid. C **36**, 2590 (1987); W.Koepf and P.Ring, Phys. Lett. B **212**, 397 (1988).

[7] N.Takigawa et al., Phys. Rev. C **50**, 1038 (1996).

[8] E.D.Cooper et al., Phys. Rev. C **47**, 297 (1993); and references cited therein.

[9] Z.Ren et al., Phys. Lett. B **380**, 241 (1996); D.Hirata et al., Nucl. Phys. A **609**, 131 (1996); J.Meng and P.Ring, Phys. Rev. Lett. **77**, 3963 (1996).

[10] T.Hatsuda, H.Shiomi and H.Kuwabara, Prog. Theor. Phys. **95**, 1009 (1996); and references cited therein.

[11] S.Nishizaki, H.Kurasawa and T.Suzuki, Nucl. Phys. A **462**, 687 (1987).

[12] A.F.Bielajew, Nucl. Phys. A **367**, 358 (1981).

[13] W.Koepf and P.Ring, Nucl. Phys. A **493**, 61 (1989); ibid. A **511**, 279 (1990).

[14] J.König and P.Ring, Phys. Rev. Lett. **71** 3079 (1993); A.V.Afanasjev, J.König and P.Ring, Phys. Lett. B **367**, 11 (1996); Nucl. Phys. A **608**, 107 (1996); J.König, Ph.D.thesis, 1996 (unpublished).

[15] K.Kaneko, M.Nakano and M.Matsuzaki, Phys. Lett. B **317**, 261 (1993).

[16] S.Weinberg, *Gravitation and Cosmology: Principles and Applications of The General Theory of Relativity* (John Willy and Sons, New York, 1972), p. 365; N.D.Birrell and P.C.W.Davies, *Quantum Fields in Curved Space* (Cambridge Univ. Press, London, 1982), p. 81.

[17] C.G.de Oliveira and J.Tiomno, Nuovo Cimento **24**, 672 (1962); T.C.Chapman and D.J.Leiter, Am. J. Phys. **44**, 858 (1976).